APPENDIX A

Binary Turbo Codes with Gray Mapping for OFDM and OFDM with MIMO Communication Systems

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1. Introduction

In a communication system using orthogonal frequency division modulation (OFDM), the transmission symbols (Tx symbols) are modulated on different subcarriers, called frequency bins. Since Tx symbols sent over different subcarriers experience different signal to noise ratio conditions, the number of bits per symbol can be sent with an acceptable error rate, are different for different subcarriers.

In an OFDM communication system with multiple transmitting and receiving antennas (i.e., multiple input/multiple output, or MIMO), the communication channel of each frequency bin can be further decomposed into multiple independent subchannels. The number of such subchannels is less than or equal to the lesser one of the number of the transmitting and the number of receiving antennas. Each of such subchannels is also called an eigen-mode if these subchannels are independent of each other. A Tx symbol can be sent over each of the eigen-modes in each frequency bin. Since the SNRs are different for each eigen modes, the number of bits can be sent over each eigen-mode is also different.

Due to the non-uniform nature of the Tx symbols over different frequency bins and eigen modes, which may be different for different system set-up and may vary with time, a powerful yet flexible coding scheme is needed for the OFDM communication systems with or without MIMO.

In recent years, Turbo code has been as the choice of error-correction coding method due to its near-optimal performance and moderate complexity. The binary form of the Turbo codes has been investigated relatively thoroughly and better understood. Such Turbo codes can be straightforwardly used when the Tx symbols are BPSK or QPSK modulated. When considering applications under high-SNR environment, which the OFDM schemes usually targeted, high-order modulations, such as 8PSK, 16QAM and 64 QAM, become necessary to increase the bandwidth efficiency.

Even though Turbo trellis coded modulation (TTCM) schemes has been considered previously by a number of researchers. A simple alternative approach is to use punctured binary turbo codes and Gray mapping for high-order modulation. As it was shown in [1], somewhat surprisingly, the latter simple scheme can achieve a performance quite close to the more optimal TTCM while provides a high degree of flexibility. Its performance is very robust under different puncture parameters. Thus, the punctured binary turbo codes

mapped to high-order modulation through Gray mapping is a good candidate for channel coding in OFDM and OFDM with MIMO systems.

2. Description

Basics of Punctured Turbo Codes with Gray Mapping

As shown in [1] *binary turbo code with Gray mapping* (BTC-GM) uses conventional binary turbo codes directly, and map the coded binary symbols onto a high-order constellation after these coded symbols being punctured to proper coding rate. A reasonable choice for the mapping is the Gray mapping. The diagram of such an encoding scheme is shown in Figure 1. We call such a scheme *binary turbo code with the Gray mapping* (BTC-GM). The encoding procedure is essentially the same as in CDMA2000 except the last step, where several binary symbols are grouped as a non-binary symbol and mapped onto a high-order constellation. Note that the mapping is carried out after the channel interleaving. Here the channel interleaver serves an additional role. It ensures that the binary symbols, which form a single non-binary symbol, are not close to each other at the output of the binary encoder. It was found, however, for static AWGN channels, such channel interleaver is less critical than for non-Turbo coded schemes due to the existence of the code interleaver.

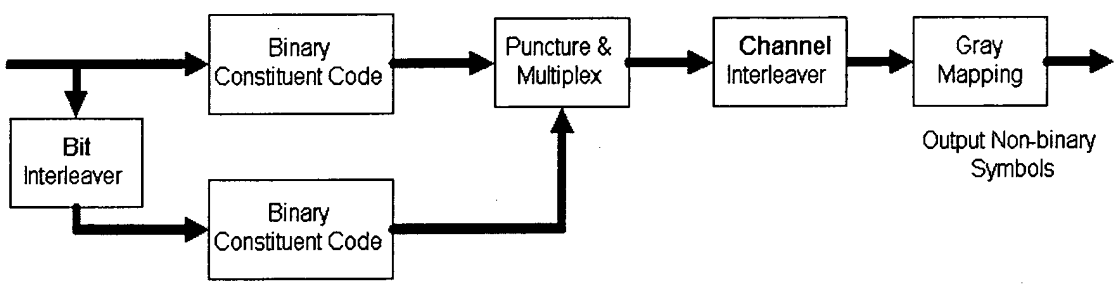

Figure 1. Coding scheme using binary turbo code with the Gray mapping.

To achieve high bandwidth efficiency for high order modulation, the punctured binary Turbo code has a relatively high rate, e.g., r = 5/6. Even though such a high rate binary code is known to require a higher E_b/N_0 than that for a lower rate binary codes, Such high rate binary codes with Gray mapping yield satisfactory performance for bandwidth efficiency coding and modulation. As shown in [1] the performance is about or less than 1.5 dB from the capacity bound for the corresponding modulation symbols.

For BTC-GM schemes that we investigated, the high rate binary Turbo code is generated by simply puncturing the non-systematic bits of the rate 1/3 Turbo code as described in the cdma2000 standard. It was found that the performance of the BTC-GM is insensitive to the specific puncturing pattern. Only two rules are used in the design the puncturer: (1) the number of bits punctured should be balanced between the two constituent codes; (2) the remaining bits should be distributed relatively evenly over the coded block. In our implementation, we arrange the parity bits from the two constituent encoders as two linear array in the order that it is generated. The remaining bits are selected from each of the arrays as uniformly as possible. The remaining parity bits and

the systematic bits are then combined in to a single bit stream, interleaved, grouped and Gray-mapped into the high order modulation symbols.

Based on the above encoding scheme, a decoder can be implemented as in Figure 2. First, the log-likelihood ratios (LLR) of the binary symbols are calculated from the received signals. The binary symbol LLR's are de-interleaved and then fed into a binary turbo decoding algorithm. The remaining decoding algorithm is exactly the same as used for conventional binary Turbo decoders.

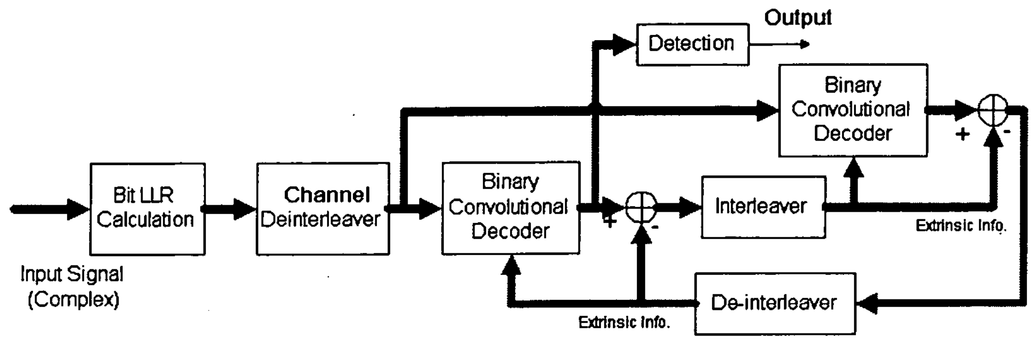

Figure 2. Decoding scheme using binary turbo code with the Gray mapping.

BTC-GM for OFDM and OFDM with MIMO

As stated above, one Tx symbol is transmitted over a frequency bin/eigen-mode. Each of such frequency-bin/eigen-mode as an SNR that is know to both transmitter and receiver. The modulation and coding parameters of the Tx symbol are determined by the SNR of the corresponding frequency-bin/eigen-mode. Below we discuss two of methods that determine the modulation/coding parameters for BTC-GM in the OFDM and OFDM with MIMO communication systems.

Both methods start with determining the modulation symbol to be used for each bin/mode by table look-up. The look-up table is generated from BTC-GM simulation as shown in [1]. One example of such a table is given in Table 1.

Table one the required Eb/No range for < 1% FER

SNR range	# of info. bits/symbol	Modulation Symbol	# of coded. bits/symbol	Coding rate
1.2 – 4.4	1	QPSK	2	1/2
4.4 – 6.4	1.5	QPSK	2	3/4
6.4 – 8.35	2	16QAM	4	1/2
8.35 – 10.4	2.5	16QAM	4	5/8
10.4 – 12.3	3	16QAM	4	3/4

12.3 – 14.15	3.5	64QAM	6	7/12
14.15 – 15.55	4	64QAM	6	2/3
15.55 – 27.35	4.5	64QAM	6	3/4
> 17.35	5	64QAM	6	5/6

The first methods uses a BTC code with a single coding rate over all bins/modes. The procedure to determine the modulation/coding parameters is as follows.

(1) Determine the modulation symbol constellation for each bin/mode based on its SNR.

(2) Compute the number of total coded bits (systematic + parity bits + tail bits) based on the result from (1).

(3) Determine the number of systematic bits (including information + systematic tail bits) for each bin/mode based on the SNR of that bin/mode.

(4) Compute the number of total systematic bits by summing what calculated above.

(5) Use 1/3 BTC to code the systematic bits. (Assume the required coding rate is greater than 1/3.

(6) Puncture the parity bits to the required number of all coded bits (systematic + parity bits + tail bits).

(7) Gray map the coded bits to the symbols determined in (1) and send these symbols over corresponding bins/modes.

The above approach is relatively simple to implement. The problem is that if the distribution of the SNRs is widespread, the distance between the constellation points relative to the noise variance in different constellation will very widely. This may impact the performance. Below we show another method to improve the normalized distance.

The second methods uses a BTC code with variable puncturing rate over bins/modes with different SNR. The procedure to determine the modulation/coding parameters is as follows.

(1) Determine the number of systematic (including information + systematic portion of tail) bits to be transmitted for each bin/mode based on its SNR. The computation is based on a table similar to Table 1 above but can have more quantization levels. The modulation symbol constellation is determined based on the number of bits per symbol.

(2) Group the bins/modes into multiple segments such that these bins/modes in a segment support the same number of systematic bits.

(3) Compute the number of the total systematic bits and total parity bits that can be transmitted in each segment, e. g., in segment i there are K_i bins/modes, each of which can support N_i systematic bits and P_i parity bits. Compute the number of total systematic bits according to $\sum_i K_i N_i$

(4) Use 1/3 BTC to code the systematic bits computed above. (Assume the required coding rate is greater than 1/3).

(5) Assign $3N_i$ (interleaved) coded bits ($2N_i$ parity bits and N_i systematic bits) to the i-th segment.

(6) Puncture the $2N_i$ parity bits to yield P_i parity bits to be transmitted.

(7) Gray map the (N_i+P_i) coded bits to the K_i symbols determined in (2) and assign these symbols to corresponding bins/modes for transmission.

By using this approach, the number of bits required to be sent over each bin/mode needed to be communicated from the receiver to the transmitter. The quantization granularity can be finer because it has a quantization level of one bit over the segment of N_i systematic bits.

The granularity can be further reduced if we can carry over the quantization from segment to segment. Namely, in the case we need to round-off one bit in one segment, we may round-up one bit in the next segment if appropriate.

A Method of Flexible Punctured (or Retaining) Parity Bits

To implement the BTC-GM for OFDM and OFDM with MIMO communication systems, it is necessary to have a flexible puncturing scheme to generate the necessary number of parity bits in each segment. Since the coding rate is usually, higher than 1/2, it may be easier to implement the puncturing by retaining a required number of bits. For example, if we have 20 16QAM symbols in a segment and the SNR allows to transmit 2.75 bits per symbol, there will be 55 systematic bits and 35 parity bits in this segment. Thus, we need to puncture the 110 parity bits to 35. We can use the following method to achieve retaining P out of Q parity bits as follows.

We construct a first counter which will wrap around after it content reaches beyond Q-1 (modulo Q operation). There will be a second counter counting the Q parity bits. Both counters are always set to zero initially. In the Q parity bits, the first bit is always to be retained. Then the counting starts by incrementing the first counter by P and the second counter by 1. If the first counter experience a modulo operating, the bit corresponding to the content of the second counter will be retained. This process continues until all of the Q bits are exhausted. A similar approach has been proposed and accepted in principle in 3GPP2 AMR flexible rate generation [2].

3. Framing Considerations

It is probably natural to consider one or a integer multiple of OFDM symbol as a physical frame. However, due to varying coding rate from one bin/mode to next, the number of information bits per such a physical frame will vary from transmission to transmission in fine steps. Thus, it may be difficult to tie such physical frames to data packets, or logical frames, which are preferably to have a small number of sizes of information bits. One possibility to solve this problem is to make the logical frame (data packets) to be independent of OFDM symbols. Namely, coding is performed over data packets. The coded bits of these packets are grouped together then mapped to OFDM symbols. Thus, a data packet can span more than one OFDM symbol and can cross the OFDM symbol boundaries. Consequently, each OFDM symbol can contained the coded bits from multiple data packets.

References:

[1] Yongbin Wei and Fuyun Ling, “Binary Turbo Codes with Gray Mapped High-Order Modulation,” Qualcomm internal memo, June 2000.

[2] “Method and Apparatus for Puncturing Code Symbols in a Communication System,” Qualcomm Patent Application Ref. No PA000334, Nov.(?) 2000.